MONROE

Machine Methods

FOR THE EXTRACTION OF SQUARE ROOT

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CALCULATING MACHINE COMPANY, INC.

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FOR THE EXTRACTION OF SQUARE ROOT

THE EXTRACTION of square root by the arithmetical, longhand process is tedious and laborious, and all too frequently subject to error.

The use of a slide rule or logarithmic tables for square root extraction is fairly fast for those well versed and experienced. The slide rule, however, is limited as to the number of digits obtainable in the result, with accuracy assured.

The method for the extraction of square root which is the most simple, speedy, and accurate regardless of the number of decimal places required, is that employing a Monroe Adding-Calculator. The Monroe method described in the following pages has been in use for many years by business firms, colleges, engineers, and statisticians.

Monroe Method Described Arithmetically and Algebraically

The Monroe machine method for extracting square root described in detail in the following pages is entirely a mechanical process. It can be learned quickly and, once learned, requires no mental effort. It has been adopted by thousands of Monroe users, particularly because of the fact that it has a direct relation to the arithmetical and algebraic principles of extracting square root.

Arithmetically

After the amount from which the root is to be extracted is placed in the Monroe machine, all operations are a matter of the subtraction of the odd numbers. For example, in finding the root of 9, when successive odd numbers are subtracted, 1, 3, and 5 are subtracted making a total of three subtractions. The sum of the three terms subtracted $(1+3+5)$ is 9, or the square of 3. Therefore 3 is the root of 9. Also the last term subtracted, 5, is one less than twice the root $(2\times3=6)$.

Thus the root is secured mechanically by subtracting the successive terms of the series. The last term subtracted, plus 1, is left on the keyboard of the machine (it is twice the root in the upper dials), and must

again be subtracted in the subsequent calculations of the problem the same number of times as the following parts of the root.

Algebraically

$(a+b)^2=a^2+(2a+b)b$

Substituting numerical values:

$(46)^2$ or $(40+6)^2=40^2+(2\times40+6)6$

Therefore when the second factor, b, (or succeeding factors) is being subtracted in a manner similar to that of the first, twice the first is also being taken out.

Monroe Automatic Subtraction Method

I Decimal Rule *Upper Dials:* Set a decimal marker in the upper dials at the number of decimal places desired in the root. *Keyboard:* Set a decimal marker on the keyboard at the same number of places as that in the upper dials. *Lower Dials:* The number of decimal places to be set off in the lower dials is the sum of those set in the upper dials plus those set on the keyboard.

II Grouping The digits in the number, the root of which is to be extracted, are grouped in pairs mentally or marked off on the machine by decimal markers. This is necessary in order to locate the column in which to start setting up the successive odd numbers. If the grouping is done on the keyboard of the machine use decimal markers to set off the groups, starting at the decimal point and counting to the left. If the number contains an odd number of digits there will be an odd number of groups; for example 23875 is grouped 02 38 75. If the number contains an even number of digits there will be an even number of groups; for example 4386 is grouped 43 86. If the number is entirely a decimal amount the figures are grouped in the same way, starting at the decimal point and counting to the right; for example .6875 is marked off .68 75.

III Keyboard Set-up Following are directions for the first set-up on the keyboard of 1, as the first odd number.

A *When the number is a whole number, or a whole number and a decimal.*

Set the 1 in that column of keys to the left of the keyboard decimal marker corresponding to the number of groups of two in the amount in the lower dials. For example, if the lower dials amount has three groups of two digits to the left of the decimal point, the 1 is set in the third row of keys to the left of the decimal marker on the keyboard. If there are two groups of two digits, set the 1 in the second column, and so on.

B *When the number is a decimal.*

Set the 1 in the first row of keys to the right of the keyboard decimal marker, provided the first group of two digits in the lower dials amount

contains significant figures. If the first group contains only ciphers, set the 1 in the second row of keys, provided the second group contains a significant figure and not two ciphers.

IV Carriage Position When the grouping into sets of two digits has been made and the first 1 set correctly on the keyboard, the carriage should be positioned as follows before starting to perform the subtraction.

A *When the amount in the lower dials is a whole number or a whole number and decimal.*

Position the carriage so the 1 already set on the keyboard is directly in line with the right-hand digit of the first group of two digits at the left in the lower dials. For example, if the first group in the lower dials amount is 26, shift the carriage so the 1 on the keyboard is in line with the 6 in the group 26. If the group is 07, position the carriage so the 1 on the keyboard aligns with the 7 in the group 07.

B *When the amount in the lower dials is a decimal.*

Position the carriage so the 1 on the keyboard is directly in line with the right-hand digit of the first group of pairs containing a significant figure and not two ciphers. For example, if the amount in the lower dials is .009312 the carriage is shifted so that the 1 on the keyboard is in direct alignment with the 3 in the lower dials.

V Subtraction Process Having positioned the carriage, the next step is the subtraction of successive odd numbers, starting with 1, to secure the root which will appear in the upper dials. The following are general instructions for performing the automatic subtraction process.

A Using two hands, set the odd numbers successively on the keyboard of the Monroe with the left hand and with the right hand depress the minus bar, once for each key set-up. For example, set with the left hand 1, 3, 5, 7, 9, 11, 13, etc., for each depressing the minus bar.

B Whenever the subtraction cannot be made a row of 9s will appear to the left of the number in the lower dials. When only one over-subtraction has been made, it is only necessary to depress the plus bar once to correct the over-subtraction.

C After the over-subtraction has been corrected, change the key last depressed to the next lower digit, which is always an even number. Shift the carriage once to the left and starting with the 1 in the next column to the right continue subtracting the odd numbers as previously described until the next over-subtraction takes place.

D In the above process if the number on the keyboard is 29, or 289, or 3419, and no over-subtraction has yet been made, the next higher odd figure for these numbers is 31, 291, and 3421. Thus it becomes necessary to change the keys set in the last two columns instead of in the last column only, as is done on most of the subtractions. If three or

more of the final digits are 9s, three or more keys will have to be changed accordingly.

E Whenever the 1 is set up and the item cannot be subtracted it is then necessary to clear the 1 by depressing the column clear key. After shifting the carriage one place, be sure to skip that row of keys in setting up the next 1.

VI General If the root has been extracted in accordance with these instructions the amount remaining on the keyboard will equal exactly twice the amount of the root in the upper dials. This check for accuracy can be made at any time during the operation.

It is possible to avoid an over-subtraction in extracting the root by carefully watching the lower dials to see whether the amount on the keyboard can be subtracted. Sometimes it is rather difficult to do this, however, particularly with large numbers, and is not nearly so mechanical or automatic.

When the root, represented by the figures in the upper dials, has been found clear the keyboard only. If it is desired to check the work, set the root on the keyboard and multiply by the amount in the upper dials. If the root is correct the upper dials will clear to zero and the lower dials will show the amount originally registered there.

In performing square root on the Monroe Adding-Calculator the repeat key must always be depressed. When using a Monroe equipped with carry-over (reversible) upper dials, the change lever should be in the ÷ position.

Examples of Automatic Subtraction Method

1 Whole Number and Decimals

$\sqrt{73083.6809}=270.3399$

Decimal set-up
(for root to four decimal places)

Upper Dials	4
Keyboard	4
Lower Dials	8

Note When the root is required to any other number of decimal places the decimal markers should be set in this same ratio.

Step 1 With the carriage in the "5" position, set 73083.6809 on the keyboard at the decimal. Depress the plus bar once. Clear keyboard and upper dials.

Step 2 An inspection of the whole number in the amount in the lower dials shows that there are three groups of two digits each: 07 30 83.

Step 3 Set 1 in the seventh column of the keyboard, which is the third row to the left of the decimal marker. Shift the carriage to the "7" position so that the 1 on the keyboard is directly in line with the 7 in the lower dials, which is the right-hand digit of the first group, 07.

Step 4 With the left hand setting keys and the right hand simultaneously depressing the minus bar, subtract successively the odd figures in the series: 1, 3, 5, etc., until an over-subtraction occurs.

Step 5 When 5 has been subtracted the over-subtraction occurs. Depress the plus bar once. Change the keyboard set-up from 5 to 4. Set 1 in the sixth column of the keyboard. Shift the carriage to "6" position. Keyboard now reads 410.00000.

Step 6 Depress odd numbered keys successively in sixth row, subtracting each one. When the 9 key is depressed and subtracted, there is still no over-subtraction and the keyboard reads 490. Change keyboard set-up to 510. and subtract. There is still no over-subtraction. Continue subtracting 530. and 550. At the latter point over-subtraction takes place. Depress the plus bar once. Change 5 in the sixth column to 4. Shift carriage to "5" position. Place 1 in fifth column. Keyboard now reads 541.0000.

Step 7 On the first subtraction in the fifth column an over-subtraction is made. Depress the plus bar once. Change the 1 key in the fifth column to 0 by depressing the column clear key. Shift carriage to "4" position and set 1 in fourth column. Keyboard now reads 540.1000 and upper dials read 270.0000.

Step 8 Continue the same procedure to subtract odd numbers. Over-subtraction takes place with the 7 in the fourth row. Correct and change the 7 to 6. Shift carriage to "3" position. Depress the 1 key in the third column. Keyboard now reads 540.6100.

Step 9 Continue this procedure until the carriage is in the "1" position. Upper dials read 270.3399, the square root of 73083.6809. Lower dials read 1936799, the remainder. Keyboard reads 540.6798, which is twice the amount of the root in the upper dials.

Step 10 To prove: Clear keyboard only. Copy to the keyboard from upper dials, 270.3399, and multiply with the plus bar by 270.3399. Upper dials clear as proof of correct multiplier; lower dials register 73083.6809, original amount placed there in Step 1, proving that the square root, 270.3399, is correct.

2 Decimal

$\sqrt{.6835}=.8267$

Decimal set-up (for root to four decimal places)	
Upper dials	4
Keyboard	4
Lower Dials	8

Step 1 With the carriage in the "5" position set .6835 on the keyboard at the decimal. Depress plus bar once. Depress master clear key and minus bar simultaneously to clear upper dials and keyboard.

Step 2 The first group to the right of the decimal in the lower dials contains significant figures. Therefore, set 1 in the fourth column on the keyboard, or first row to the right of the decimal. Move carriage to the "4" position so that the 1 on the keyboard aligns with 8, or right-hand digit of the first group, 68.

Step 3 As explained above, subtract odd numbers. Over-subtraction takes place when the keyboard reads 1.7000. Correct to 1.6000. Upper dials read .8000. Move carriage to "3" position and set 1 in the third row on the keyboard. The amount on the keyboard is now 1.6100.

Step 4 Continue the same procedure until the carriage reaches the "1" position. Upper dials read .8267, the square root. Lower dials register 6711, the remainder. The amount on the keyboard, 1.6534, is exactly twice the root, .8267, in the upper dials.

Step 5 To prove: Clear keyboard only. Set .8267 on the keyboard. Multiply by .8267 with plus bar. Upper dials clear to zero, proving correct multiplier was used. Lower dials read .6835, original amount placed in lower dials in Step 1, proving the root, .8267, is correct.

Monroe Automatic Inspection Method

I Decimal Rule Same as for Automatic Subtraction Method.

II Grouping Same as for Automatic Subtraction Method.

III Carriage Position

A *Whole Number and Whole Number and Decimals*

Starting with the carriage positioned so the keyboard and lower dials decimal markers align, shift the carriage to the right to the position corresponding to the number of groups in the amount in the lower dials to the left of the decimal point. For example, if the amount in the lower dials is 642.68, there are two groups to the left of the decimal point, 06 42. Therefore, the carriage is moved one place to the right so it is in the second position at the left of the upper dials decimal.

B *Decimals*

If the lower dials amount is a decimal, position the carriage so the keyboard and lower dials decimal markers align. Then shift the carriage one place to the left, if the first group of digits in the amount in the lower dials contains significant figures. For example, if the amount in the lower dials is .4285, the first group contains significant figures; therefore the carriage should be shifted to the left one place, or to the first position to the right of the upper dials decimal point.

IV Inspection Process With the carriage positioned, proceed as follows.

A Estimate what number will produce the nearest square less than the amount of the first two digits in the first group. Set that number on the keyboard directly under the right-hand figure of the first group and subtract the estimated number of times.

B Mentally double the amount on the keyboard and change the keyboard set-up to that figure. Shift the carriage one place to the left.

C Estimate the number of times the keyboard amount will go into the lower dials amount directly in line with the keyboard figures. Set the estimated number on the keyboard in the next clear column to the right of the keyboard amount and subtract the estimated number of times.

D Increase the keyboard amount by an amount equal to the right-hand digit of the keyboard set-up. Shift carriage one place to the left.

E Again estimate the number of times the keyboard amount will go into the lower dials amount that is directly in line. Set that number on the keyboard in the next clear column to the right and subtract the amount that number of times. Increase the keyboard amount by an amount equal to the last right-hand digit on the keyboard. Shift carriage to left one place and continue same procedure until completion of the problem when the carriage is in the "1" position.

Example of Automatic Inspection Method

$\sqrt{642.68}=25.3511$

Decimal set-up
(for root to four decimal places)

Upper Dials	4
Keyboard	4
Lower Dials	8

Step 1 With the carriage in "5" position, set 642.68 on keyboard at decimal. Depress plus bar once. Depress master clear key and minus bar simultaneously.

Step 2 By inspection it is found there are two groups in the lower dials amount to the left of the decimal, 06 42. Therefore the carriage

is shifted to "6" position which is the second place to the left of upper dials decimal.

Step 3 The first group is 06. It is noted that 2 will produce the nearest square to 6; therefore set 2 on the keyboard in the sixth column or directly under the 6 in the lower dials, and depress the minus bar twice.

Step 4 Change keyboard amount to 4, which is double the 2, so the keyboard reads 40.0000. Shift carriage one place to the left or to "5" position. It is estimated 4 on the keyboard will go into the 24, which is directly above it in the lower dials, approximately 5 times. Set 5 in fifth column so keyboard reads 45.0000. Subtract five times. Lower dials now read 17.68000000.

Step 5 Increase 45. on keyboard by the amount of the last digit, 5, changing keyboard set-up to 50.0000. Shift carriage to "4" position.

Step 6 An estimation indicates 5 will go into the 17 in the lower dials 3 times. Depress the 3 key in fourth column of keyboard, making keyboard set-up 50.3000. Subtract three times. Lower dials now read 2.59.

Step 7 Increase 50.3 on keyboard by the amount of last digit, 3, so keyboard set-up is 50.6. Shift carriage to "3" position.

Step 8 It is estimated that 5 will go into 25 five times so set 5 in the third column on the keyboard; keyboard reads 50.65. Subtract five times. Lower dials read .0575.

Step 9 Continue same procedure for rest of problem. Upper dials read 25.3511, the square root; lower dials, 172879, remainder; keyboard, 50.7022. The latter amount is exactly twice the root, 25.3511, in upper dials.

Step 10 To prove: Clear keyboard only. Copy 25.3511 from upper dials to keyboard. Square by multiplying. Upper dials clear as proof that correct multiplier was used. Lower dials read 642.68, amount placed in lower dials in Step 1, as proof that 25.3511 is the correct square root.

Monroe Automatic Subtraction and Division Method

I Decimal Rule Same as for Automatic Subtraction Method.

II Grouping Same as for Automatic Subtraction Method.

III Carriage Position Same as for Automatic Subtraction or Inspection Method.

IV Division Process

A This method can be used whether the square root is being extracted by the Automatic Subtraction Method or the Automatic Inspection Method. Either of these methods must first be used before the Automatic Subtraction and Division Method can be started.

B After starting to extract the square root by either of the two methods, it is possible to finish the problem by automatic division.

C The division process can be used only when the number of unfilled digits in the upper dials is one less than the number of digits that are filled. For example, if there are four decimal places in the upper dials and the dials which contain figures secured by the subtraction or inspection process read 243.4000, there are three unfilled dials and four dials with figures. In other words, the number of unfilled dials is one less than the number of filled dials so it is then possible to continue by dividing the amount in the lower dials by the amount remaining on the keyboard, which will carry out the root for three more places.

Example of Automatic Subtraction and Division Method

$\sqrt{46032.5806}=214.5520$

Decimal set-up
(for root to four decimal places)

Upper Dials	4
Keyboard	4
Lower Dials	8

Step 1 Using the Automatic Subtraction Method as previously described, extract the root up to and including the "4" carriage position. Upper dials then read 214.5000; lower dials, 22.33060000; keyboard, 429.0100. As an inspection of the upper dials shows there are four with figures and three with zeros, automatic division can be used to secure the last three digits of the root.

Step 2 Shift the carriage to the "3" position and divide. The amount in the upper dials, 214.5520, is the square root. The lower dials read .02208000, remainder. The amount on the keyboard is 429.0100, which in this method is not twice the root.

Step 3 To prove: Clear keyboard only. Copy to keyboard the upper dials amount, 214.5520. With the plus bar square. Upper dials clear, proving that correct multiplier was used. Lower dials read 46032.58278400, which is approximately the amount that was set in the lower dials in Step 1, 46032.5806.

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Monroe methods for extracting cube and higher roots are explained in detail in a separate booklet, Form 720-S

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For further instructions or help on figuring problems, Monroe users are invited to get in touch with any of the Branch Offices which are located in all principal cities. Monroe representatives are qualified to recommend the most efficient applications for the handling of figure work. A methods department at the General Offices in Orange, New Jersey, also welcomes the opportunity to be of service to Monroe users.